SEC. 9.2

PRE CALC Class examples PARABOLAS NAME _____ MOD __

EXAMPLE 1: FIND the focus and directrix of each parabola whose equation is given.

(a) $y = 2x^2$ focus = $(0, \frac{1}{8})$ directrix = $y = -\frac{1}{8}$

(b) $x = \frac{1}{20}y^2$ focus = $(5, 0)$ directrix = $x = -5$

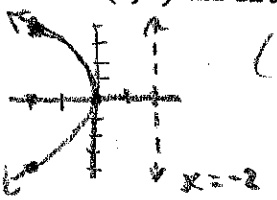
EXAMPLE 2: FIND an equation of the parabola with vertex (0, 0) and directrix x = 2.

$y^2 = -8x^2$

$(y-0)^2 = 4p(x-h)$

$(y-0)^2 = 4(-2)(x-0)$

$p = -2$ b/c the Focus is (-2) units from the vertex



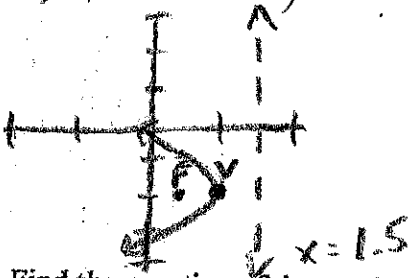
$(y-k)^2 = 4p(x-h)$

$(y-0)^2 = 4(-2)(-2-0)$

$y^2 = 16 \Rightarrow y = \pm 4$

EXAMPLE 3: Tell whether the parabola $x - 1 = -\frac{1}{2}(y+2)^2$ opens up, down, left or right. Give the coordinates of the vertex and focus and the equation of the directrix. Sketch the graph. opens left

vertex = $(1, -2)$ focus = $(\frac{1}{2}, -2)$ directrix = $x = 1.5$



$-2(x-1) = (y+2)^2$

$(y+2)^2 = -2(x-1)$

$4p = -2$

$p = -\frac{1}{2}$

$|p| = \frac{1}{2}$

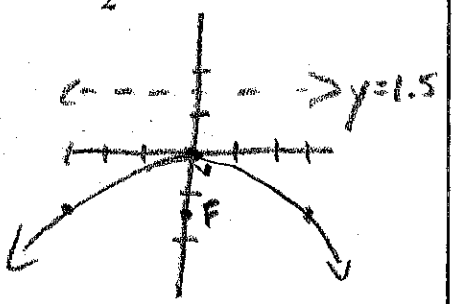
EXAMPLE 4: Find the equation of the parabola with vertex (0, 0) and focus $(0, -\frac{3}{2})$. Sketch the graph.

$x^2 = -6y$

$(x-0)^2 = 4p(y-0)$

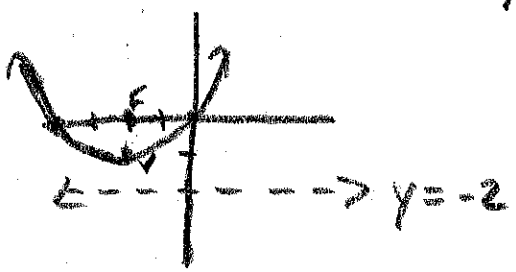
$(x-0)^2 = 4(-\frac{3}{2})(y-0)$

$p = -\frac{3}{2}$



EXAMPLE 5: FIND the vertex, focus, and directrix of the parabola $x^2 + 4x - 4y = 0$. Graph the equation.

vertex = $(-2, -1)$ focus = $(-2, 0)$ directrix = $y = -2$



$x^2 + 4x + \boxed{4} = 4y + \boxed{4}$

$(x+2)^2 = 4(y+1)$

$4p = 4 \rightarrow \boxed{p = 1}$

$LR = 4$

WS#9-2B
Parabolas

Identify the Axis of symmetry, vertex, focus, & directrix

1. ~~Transform the following into a recognizable form:~~

A. $(y-3)^2 = 8(x+2)$
$V(-2,3)$ $4p=8$ $p=2$
Axis of Sym: $y=3$
$F(h+p,k) \rightarrow F(0,3)$
$D: x = h-p \rightarrow x = -2-2 \rightarrow x=-4$

B. $(x+3)^2 = -4(y+2)$
$4p=-4 \rightarrow p=-1$
$V(-3,-2)$, Axis of sym $\rightarrow x=-3$
$F(h,k+p) \rightarrow F(-3,-3)$
$D: y = k-p \rightarrow y = -2-(-1)$ $y=-1$

C. $x^2-4x=2y$
$x^2-4x+4 = 2y+4$
$(x-2)^2 = 2y+4$
$(x-2)^2 = 2(y+2)$
$4p=2 \rightarrow p=\frac{1}{2}$
$V(2,-2)$
Axis of sym $\rightarrow x=2$
$F(h,k+p) \rightarrow F(2,-1.5)$
Directrix: $y = k-p = -2-\frac{1}{2}$
$y=-2.5$

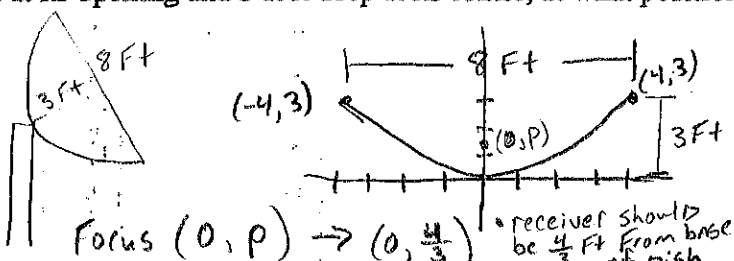
2. Graph the following on your calculator and make an accurate sketch below showing critical aspects of each:

A. $y^2 = 8x$ $4p=8$ $p=2$
$(y-0)^2 = 8(x-0)$
$h=0$ $k=0$
$V(0,0)$
$F(2,0)$
$D: x=-2$
pts that make up LR: $(2,4)$ $(2,-4)$ since $LR = |4p| = 8$

B. $x^2+6x-4y+1=0$
$x^2+6x+\boxed{9} = 4y-1+9$
$(x+3)^2 = 4y+8$
$(x+3)^2 = 4(y+2)$
$V(-3,-2)$
$F(-3,-1)$
$4p=4$ $p=1$
$LR = |4p| = 4$
$D: y=-3$
pts that make up LR: $(-5,-1)$ & $(-1,-1)$

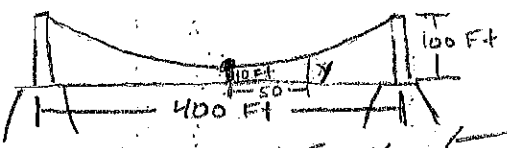
C. $y^2-2y = 8x-1$ $4p=8$ $p=2$
$y^2-2y+\boxed{1} = 8x-1+1$
$(y-1)^2 = 8x \rightarrow (y-1)^2 = 8(x-0)$
$V(0,1)$
$F(2,1)$
$D: x=-2$
pts that make up LR: $(2,5)$ & $(2,-3)$
$LR = |4p| = 8$

3. A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 8 feet across at its opening and 3 feet deep at its center, at what position should the receiver be placed?



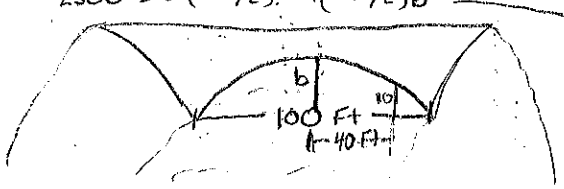
Form of parabola: $(x-h)^2 = 4p(y-k)$
$(x-0)^2 = 4p(y-0) \rightarrow x^2 = 4py$
• Since $(4,3)$ is a point on the graph, plug in for x & y AND solve for p.
$(4)^2 = 4p(3)$ $16 = 12p$ $(p = \frac{4}{3})$
Focus $(0,p) \rightarrow (0,\frac{4}{3})$
• receiver should be $\frac{4}{3}$ Ft from base of dish

#68 4. The cables of a suspension bridge are in the shape of a parabola. The towers supporting the cable are 400 feet apart and 100 feet high. If the cables are at a height of 10 feet midway between the towers, what is the height of the cable at a point 50 feet from the center of the bridge?



$V(0,10)$
Form of a parabola: $(x-h)^2 = 4p(y-k)$
$(x-0)^2 = 4p(y-10)$
• Since $(200,100)$ is a pt on the graph, plug in for x & y AND solve for p.
$(200-0)^2 = 4p(100-10) \rightarrow 40{,}000 = 360p$
$\frac{1000}{9} = p$
Sub 50 in for x & solve for y.
$(50)^2 = \frac{4000}{9}(y-10) \rightarrow y = 15.625$ Ft
$(x-0)^2 = \frac{4000}{9}(y-10)$

#74 5. A bridge is to be built in the shape of a parabolic arch and is to have a span of 100 feet. The height of the arch a distance of 40 feet from the center is to be 10 feet. Find the height of the arch at its center.



(2) To find b, sub $p = -45$ back into either of the two equations & solve for b.
$2500 = 0(-45/2) - 4(-45/2)b$
$\frac{2500}{90} = \frac{90b}{90} \rightarrow b = 27.78$ ft (1)
$(0,b) \rightarrow$ Vertex
$(40,10)$ is a pt on graph so sub in
$(40-0)^2 = 4p(10-b)$

Form of a parabola:
$(x-h)^2 = 4p(y-k)$
$(x-0)^2 = 4p(y-b)$
$(50-0)^2 = 4p(0-b)$ ($(50,0)$ is a pt on the parabola)
$2500 = 0p - 4pb$
$-1600 = 40p - 4pb$
$900 = -40p \rightarrow p = -\frac{45}{2}$